Dynamic Programming (DP)

Multistage Graph (MG)

A multistage graph $G = (V, E)$ is a directed graph in which the vertices are partitioned into $K \geq 2$ disjoint sets V_i, $1 \leq i \leq k$, if $\langle u, v \rangle$ is an edge in E, then $u \in V_i$ and $v \in V_{i+1}$ for some i, $1 \leq i < K$. The sets V_1 and V_K are such that $|V_1| = |V_K| = 1$. Let s and t, respectively, be the vertices in V_1 and V_K. The vertex s is the source, and t the sink. Let $c[i, j]$ be the cost of edge $\langle i, j \rangle$.

The multistage graph problem is to find a minimum-cost path from s to t.

There are two approaches for finding minimum-cost path in multistage graph.

(a) Forward approach

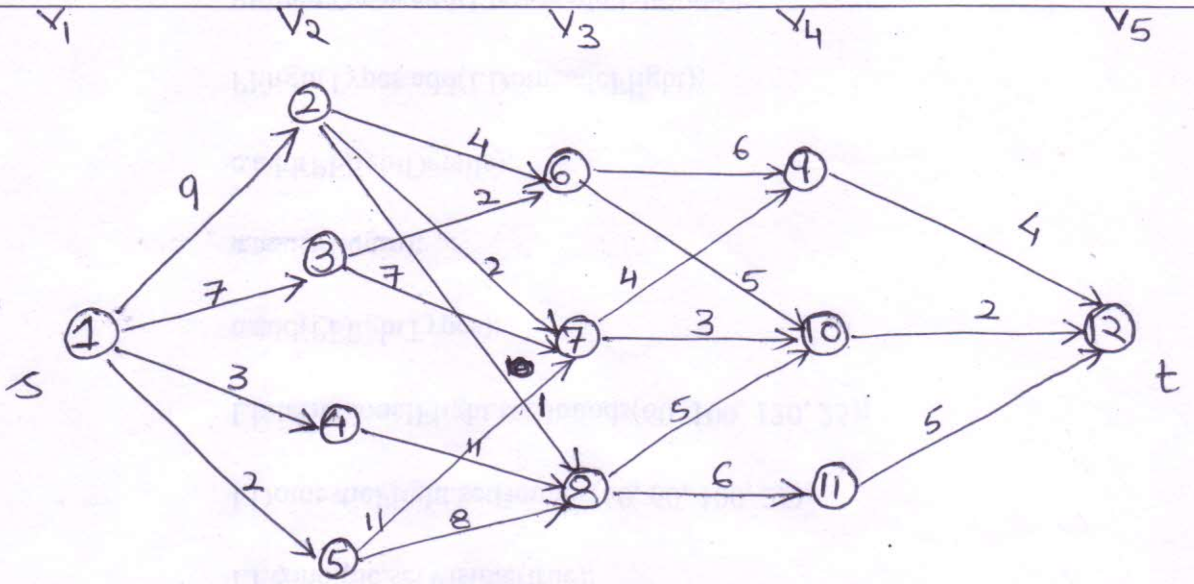
(b) Backward approach.

Forward Approach.

```
FGraph(G, K, n, p)
/* The input is a k-stage graph G=(V,E)
with n vertices indexed in order of stages.
E is a set of edges and c[i,j] is the
cost of <i,j>. p[1: k] is a minimum-
cost path. K represents no. of stages.

cost[n] = 0.0 , d[n] = n.
for j = n-1 to 1 step -1 do
{ // compute cost[j] and d[j]
  Let r be a vertex such that <j,r>
  is an edge of G and c[j,r] + cost[r]
  is minimum.
  cost[j] = c[j,r] + cost[r]
  d[j] = r.
}
p[1] = 1 , p[k] = n
for j = 2 to k-1
{ p[j] = d[p[j-1]] }
```

MULTISTAGE GRAPH



	cost	d
1	16	2/3
2	7	7
3	9	6
4	18	8
5	15	8
6	7	10
7	5	10
8	7	10
9	4	12
10	2	12
11	5	12
12	0	12

	p
1	1
2	2
3	7
4	10
5	12

	p
1	1
2	3
3	6
4	10
5	12

Final cost = 16

Two optimal paths

Path $1 \rightarrow 2 \rightarrow 7 \rightarrow 10 \rightarrow 12$.

Path $1 \rightarrow 3 \rightarrow 6 \rightarrow 10 \rightarrow 12$

Backward Approach

```
BGraph(G, K, n, p)

1  bcost[1] = 0.0,  d[1] = 1
2  for j = 2 to n do
3  { Let r be such that <r, j> is an
     edge of G and  bcost[r] + c[r, j]
     is minimum.
     bcost[j] = bcost[r] + c[r, j]
     d[j] = r
4  }
5  // Find a minimum-cost path.
6  p[1] = 1,   p[K] = n
7  for j = K-1 to 2
8  { p[j] = d[p[j+1]] }
```

The complexity is $\Theta(|V| + |E|)$

MULTISTAGE GRAPH

	bcost	d
1	0	1
2	9	1
3	7	1
4	3	1
5	2	1
6	9	3
7	11	2
8	10	2/5
9	15	6/7
10	14	6/7
11	16	8
12	16	10

	p
1	1
2	3
3	6
4	10
5	12

1
2
7
10
12

Path $12 \to 10 \to 6 \to 3 \to 1$ / $12 \to 10 \to 7 \to 2 \to 1$

Cost 16 $= 16$