STRASSEN'S MATRIX MULTIPLICATION

Conventional matrix multiplication algorithm gives $O(n^3)$ complexity but strassen's reduced it to $O(n^{2.81})$.

Let A and B are two nxn matrices. The product matrix C = AB is also an nxn matrix whose i^{th} and j^{th} element

$$C(i,j) = \sum_{1 \leq K \leq n} A(i,K)B(K,j)$$

for $n = 2$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$C_{11} = A_{11}B_{11} + A_{12}B_{21}$$
$$C_{12} = A_{11}B_{12} + A_{12}B_{22} \qquad \text{— } (Y)$$
$$C_{21} = A_{21}B_{11} + A_{22}B_{21}$$
$$C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

It takes 8 mutiplications, and 4 addition, the basic operation is addition, to add two $\frac{n}{2} \times \frac{n}{2}$ matrices takes time cn^2 for some constant c, the overall computation time T(n) of the resulting divid-and-conquer algorithm is given by recurrence

$$T(n) = \begin{cases} b & n \leq 2 \\ 8T(\frac{n}{2}) + cn^2 & n > 2 \end{cases}$$

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Strassen has reduced total multiplications from 8 to 7, but additions and subtractions are required 18, we know addition or subtraction requires less time than multiplication the overall time is reduced to $O(n^{2.81})$

Strassen's way for multiplication.

$$P = (A_{11} + A_{22})(B_{11} + B_{22})$$

$$Q = (A_{21} + A_{22}) B_{11}$$

$$R = A_{11}(B_{12} - B_{22})$$

$$S = A_{22}(B_{21} - B_{11})$$

$$T = (A_{11} + A_{12}) B_{22}$$

$$U = (A_{21} - A_{11})(B_{11} + B_{12})$$

$$V = (A_{12} - A_{22})(B_{21} + B_{22})$$

$$C_{11} = P + S - T + V$$

$$C_{12} = R + T$$

$$C_{21} = Q + S$$

$$C_{22} = P + R - Q + U$$

The recurrence relation for strassen's

$$T(n) = \begin{cases} b & n \leq 2 \\ 7T(\frac{n}{2}) + cn^2 & n > 2 \end{cases}$$

by Master's theorem

$$T(n) = O(n^{\log_2^7}) \simeq O(n^{2.81})$$